Are the chiral based $\overline{K}N$ potentials really energy-dependent ?

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The $\Lambda(1405)$ is one of the basic objects of strangeness nuclear physics.

Experimentally: a well-pronounced bump in the $\pi\Sigma$ missing mass spectrum in various reactions, just below the $K^- p$ threshold. PDG:

$$E - i\frac{\Gamma}{2} = (1405 - 25i)\,MeV$$

Theoretically: an $I = 0$ quasi-bound state in the $\bar{K}N - \pi\Sigma$ system, which decays into the $\pi\Sigma$ channel

Constructing any multichannel $\bar{K}N$ interaction – more or less reproducing the scarce and old experimental data – one of the first questions is : “What kind of $\Lambda(1405)$ it produces?”

At present it is believed, that theoretically substantiated $\bar{K}N$ interactions (potentials) – apart from the phenomenological ones – can be derived from the chiral perturbation expansion of the SU(3) meson-baryon Lagrangian.

For these interactions the widely accepted answer to the above question is, that the observed $\Lambda(1405)$ is the result of interplay of two T-matrix poles.

The subject of the talk is to challenge this opinion

Starting point

Lowest order Weinberg-Tomozawa (WT) meson-baryon interaction term of the chiral SU(3) Lagrangian (from the basic paper E. Oset, A. Ramos **NPA** 635(1998) 99):

$$\langle q_i | v_{ij} | q_j \rangle \sim -\frac{c_{ij}}{4 f_\pi^2}(q_i^0 + q_j^0)$$

Multichannel interaction, isospin basis, the channels:

$$i = 1,2,3,4,5 \Rightarrow \left[\overline{K}N\right]^{I=0}, \left[\overline{K}N\right]^{I=1}, [\pi\Sigma]^{I=0}, [\pi\Sigma]^{I=1}, [\pi\Lambda]^{I=1}$$

q_i – meson c.m. momentum

q_i^0 – meson c.m. energy $q_i^0 = \sqrt{m_i^2 + q_i^2}$; $m_i(M_i)$ – meson(baryon) masses

c_{ij} – SU(3) Clebsch-Gordan coefficients

f_π – pion decay constant

Dynamical framework:

(a) relativistic: BS equation, relativistic kinematics

(b) non-relativistic: LS equation, non-relativistic kinematics

Our choice is (b), having in mind application for N>2 systems

In practical calculations the original interaction is used with certain modifications:

– normalization + rel. correction to meson energies

$$\langle q_i | v_{ij} | q_j \rangle = -\frac{c_{ij}}{64\pi^3 F_i F_j} \frac{1}{\sqrt{m_i m_j}} (q_i^{0'} + q_j^{0'})$$

$$q_i^{0'} = q_i^0 + \frac{(q_i^0)^2 - m_i^2}{2M_i} = q_i^0 + \frac{q_i^2}{2M_i} \underset{\text{nonrel}}{\approx} m_i + \frac{q_i^2}{2\mu_i} = \gamma_i(q_i)$$

$\mu_i = m_i M_i / (m_i + M_i)$ reduced mass in channel i

F_i, $i = \left(\overline{K}, \pi\right)$ - meson decay constants

– regularization, using separable potential representation with suitable cut-off factors $u_i(q_i)$ ensuring the convergence of the integrals

Finally, the potential V_{ij} entering the LS equation

$$\left\langle q_i \middle| T_{ij}(W) \middle| q_j \right\rangle = \left\langle q_i \middle| V_{ij} \middle| q_j \right\rangle + \sum_s \int \left\langle q_i \middle| V_{is} \middle| q_s \right\rangle G_s(q_s;W) \left\langle q_s \middle| T_{sj}(W) \middle| q_j \right\rangle dq_s$$

has the form

$$\left\langle q_i \middle| V_{ij} \middle| q_j \right\rangle = u_i(q_i) \left\langle q_i \middle| v_{ij} \middle| q_j \right\rangle u_j(q_j) = \lambda_{ij}(g_{iA}(q_i) g_{iB}(q_j) + g_{iB}(q_i) g_{iA}(q_j))$$

which is a two-term multichannel separable potential with form-factors

$$g_{iA}(q_i) = u_i(q_i); \;\; g_{iB}(q_i) = u_i(q_i)\gamma_i(q_i)$$

and coupling constant

$$\lambda_{ij} = -\frac{c_{ij}}{64\pi^3 F_i F_j} \frac{1}{\sqrt{m_i m_j}}$$

The non-relativistic propagator G_s has the form

$$G_s(q_s;W) = (W - m_s - M_s - \frac{q_s^2}{2\mu_s} + i\varepsilon)^{-1} = \frac{2\mu_s}{(k_s^2 - q_s^2 + i\varepsilon)},$$

where $k_s = \sqrt{2\mu_s(W - m_s - M_s)}$ is the on-shell c.m. momemtum in channel s.

In order to simplify the solution of the dynamical equation -- both in (a) and (b) approaches -- a commonly used approximation is to remove the inherent q - dependence of the interaction, by replacing q_i in $\gamma_i(q_i)$ by its on-shell value k_i :

$$\gamma_i(q_i) \Rightarrow \gamma_i(k_i) = W - M_i$$

This is the s.c. on-sell factorization approximation and as its result, the coupling constant λ_{ij} acquires the familiar energy-dependent factor $(2W - M_i - M_j)$, which turns out to be responsible for the appearance of a second pole in the $\bar{K}N - \pi\Sigma$ system.

The multichennel two-term separable form of the potential allows an exact solution of the LS equation both with the full and on-shell factorized WT interactions thus offering a possibility to check the validity and/or consequences of the on-shell factorization.

Some technical details of the solution:

Introducing the concise matrix notations for the momenta and form-factors:

$$\left|\mathbf{q}(\mathbf{k})\right\rangle = \begin{pmatrix} \left|q_1(k_1)\right\rangle & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \left|q_n(k_n)\right\rangle \end{pmatrix}; \left|\mathbf{g}_{\mathbf{A(B)}}\right\rangle = \begin{pmatrix} \left|g_{1A(B)}\right\rangle & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \left|g_{nA(B)}\right\rangle \end{pmatrix}$$

and also for the propagator

$$\mathbf{G}(W) = \begin{pmatrix} G_1(q_1;W) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & G_n(q_n;W) \end{pmatrix}$$

the ij matrix element our potential can be written as $\left(\left\langle\mathbf{q}\right|\mathbf{V}\left|\mathbf{q}\right\rangle\right)_{ij}$ with

$$\mathbf{V} = \left|\mathbf{g_A}\right\rangle \boldsymbol{\lambda} \left\langle\mathbf{g_B}\right| + \left|\mathbf{g_B}\right\rangle \boldsymbol{\lambda} \left\langle\mathbf{g_A}\right|$$

(Bold face letters denote $n\times n$ matrices, n - number of channels).

The T - matrix has the form :

$$\mathbf{T} = \left|\mathbf{g_A}\right\rangle \boldsymbol{\tau}_{\mathbf{AA}} \left\langle\mathbf{g_A}\right| + \left|\mathbf{g_A}\right\rangle \boldsymbol{\tau}_{\mathbf{AB}} \left\langle\mathbf{g_B}\right| + \left|\mathbf{g_B}\right\rangle \boldsymbol{\tau}_{\mathbf{BA}} \left\langle\mathbf{g_B}\right| + \left|\mathbf{g_B}\right\rangle \boldsymbol{\tau}_{\mathbf{BB}} \left\langle\mathbf{g_B}\right|,$$

where the $\boldsymbol{\tau}$ matrices are $n \times n$ submatrices of the $2n \times 2n$ matrix M^{-1}

$$M^{-1} = \begin{pmatrix} \boldsymbol{\lambda}^{-1} - \langle \mathbf{g_B} | \mathbf{G}(W) | \mathbf{g_A} \rangle & -\langle \mathbf{g_B} | \mathbf{G}(W) | \mathbf{g_B} \rangle \\ -\langle \mathbf{g_A} | \mathbf{G}(W) | \mathbf{g_A} \rangle & \boldsymbol{\lambda}^{-1} - \langle \mathbf{g_A} | \mathbf{G}(W) | \mathbf{g_B} \rangle \end{pmatrix}^{-1} = \begin{pmatrix} \boldsymbol{\tau}_{\mathbf{AB}} & \boldsymbol{\tau}_{\mathbf{AA}} \\ \boldsymbol{\tau}_{\mathbf{BB}} & \boldsymbol{\tau}_{\mathbf{BA}} \end{pmatrix}$$

The convergence of all Green's function matrix elements can be ensured, if the cut-off factors $u_i(q)$ are of the s. c . dipole type:

$$u_i(q) = \left(\frac{\beta_i^2}{q^2 + \beta_i^2} \right)^2$$

In order to understand the nature of the on-shell factorization approximation, let us consider one of these matrix elements, containing $\left|\mathbf{g_B}\right\rangle$:

$$\left(\mathbf{G_{BA}}\right)_{ij} = \left(\left\langle \mathbf{g_B} \middle| \mathbf{G} \middle| \mathbf{g_A} \right\rangle\right)_{ij} = \delta_{ij} \, 2\mu_i \int \frac{u_i(q)^2 \gamma_i(q)}{k_i^2 - q^2 + i\varepsilon} d\vec{q}$$

On-shell factorization means, that $\gamma_i(q)$ is replaced by $\gamma_i(k_i) = W - M_i$ and taken out from the integral. It can be seen, that for real positive k_i , when the integrand is singular, this might have a certain justification, however, for complex k_i , which is the case, when complex pole positions are sought, the approximation seems to be meaningless.

Performing this operation for all matrix elements, the on-shell T- matrix can be written as:

$$\langle \mathbf{k}|\mathbf{T}|\mathbf{k}\rangle = \langle \mathbf{k}|\mathbf{g}_\mathbf{A}\rangle \boldsymbol{\tau} \langle \mathbf{g}_\mathbf{A}|\mathbf{k}\rangle$$

with

$$\boldsymbol{\tau} = \boldsymbol{\tau}_\mathbf{AA} + \boldsymbol{\tau}_\mathbf{AB}\boldsymbol{\gamma} + \boldsymbol{\gamma}\boldsymbol{\tau}_\mathbf{BA} + \boldsymbol{\gamma}\boldsymbol{\tau}_\mathbf{BB}\boldsymbol{\gamma}.$$

Here we introduced the matrix of on-shell γ functions

$$\boldsymbol{\gamma} = \begin{pmatrix} \gamma_1(k_1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma_n(k_n) \end{pmatrix}$$

and used the fact, that on-shell we have $\langle \mathbf{k}|\mathbf{g}_\mathbf{B}\rangle = \langle \mathbf{k}|\mathbf{g}_\mathbf{A}\rangle \boldsymbol{\gamma}$. It can be shown, that $\boldsymbol{\tau}$ can be written as

$$\boldsymbol{\tau} = \left(\left(\boldsymbol{\lambda}\boldsymbol{\gamma} + \boldsymbol{\gamma}\boldsymbol{\lambda} \right)^{-1} - \mathbf{G}_\mathbf{AA} \right)^{-1},$$

which coincides with the corresponding $n \times n$ $\boldsymbol{\tau}$ - matrix of the on-shell factorized potential $\mathbf{U}$:

$$\mathbf{U} = |\mathbf{g_A}\rangle(\boldsymbol{\lambda\gamma} + \boldsymbol{\gamma\lambda})\langle\mathbf{g_A}|$$

Now we can compare the results obtained from the "full" WT potential

$$\mathbf{V} = |\mathbf{g_A}\rangle\boldsymbol{\lambda}\langle\mathbf{g_B}| + |\mathbf{g_B}\rangle\boldsymbol{\lambda}\langle\mathbf{g_A}|$$

and its on-shell factorized, energy-dependent counterpart $\mathbf{U}$.

Both potentials depend on the same set of 7 adjustable parameters

$$F_\pi, F_{\overline{K}}, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5,$$

which have to be determined by fitting to experimental data. But before proceeding to the discussion of fit results we make our most important statement:

For any reasonable combination of the parameters, the "full" WT potential $\mathbf{V}$ produces only one pole below and close to the KN thershold, which can be associated with the $\Lambda(1405)$, while $\mathbf{U}$ produces the familiar two poles: one close to the threshold and a second one, much lower and broader.

The potential parameters were fitted to the available experimental data:
six low-energy $K^- p$ cross sections and three threshold branching ratios

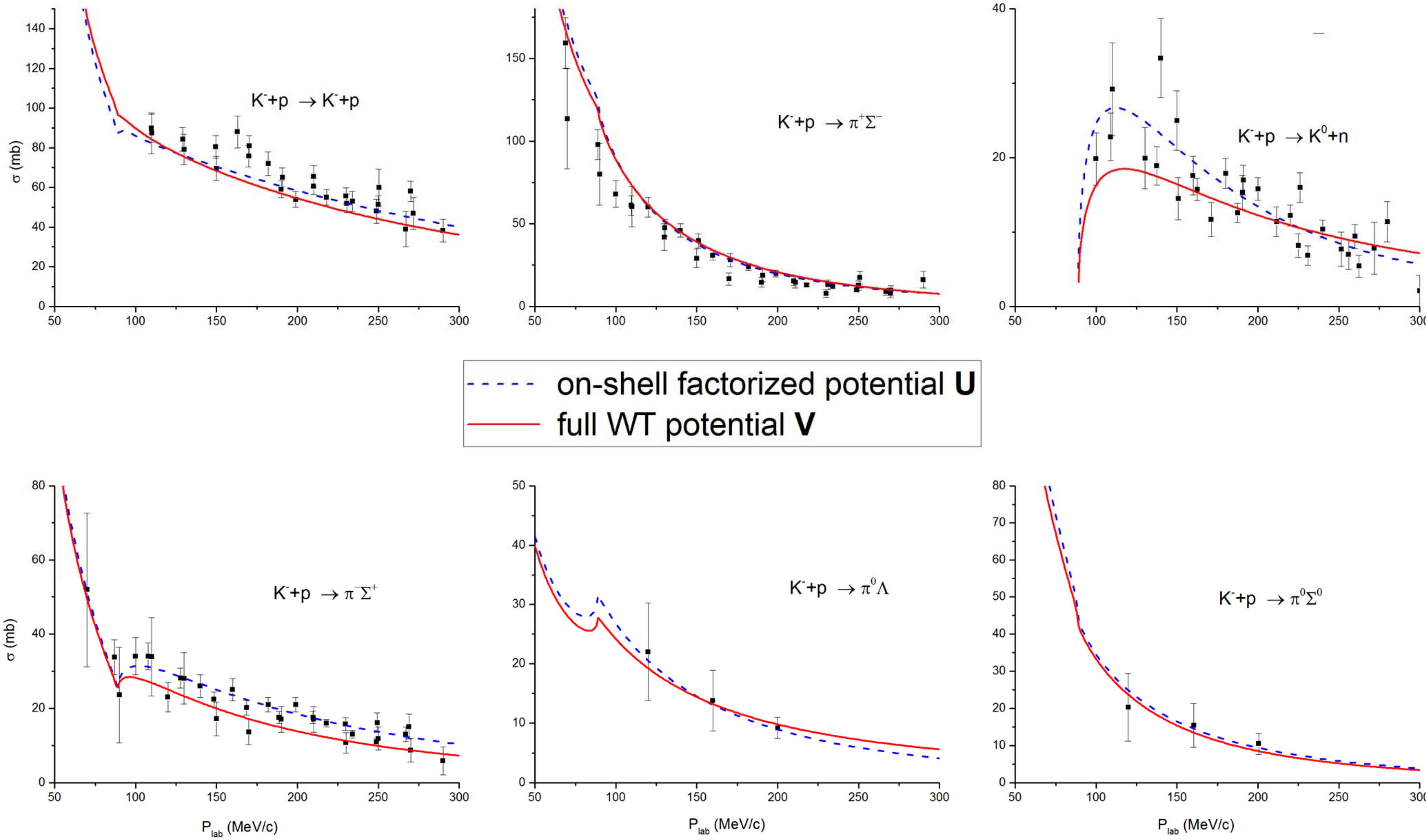
$$\gamma = \frac{\sigma(K^- p \to \pi^+ \Sigma^-)}{\sigma(K^- p \to \pi^- \Sigma^+)};\ R_n = \frac{\sigma(K^- p \to \pi^0 \Lambda)}{\sigma(K^- p \to \pi^0 \Lambda, \pi^0 \Sigma^0)};$$

$$R_c = \frac{\sigma(K^- p \to \pi^+ \Sigma^-, \pi^- \Sigma^+)}{\sigma(K^- p \to \text{all inelastic channels})}$$

and the $1s$ level shift ΔE in kaonic hydrogen.

Results of the fit:

Low-energy cross sections:



Branching ratios and ΔE :

	Υ	R_c	R_n	$\Delta E(eV)$
U	2.35	0.664	0.194	302 - 294 ***i***
V	2.32	0.671	0.202	350 – 279 ***i***
Exp	2.36±0.04	0.664±0.011	0.189±0.015	(283±36) - (271±46) ***i***

The obtained parameter values (in MeV) :

	F_π	F_K	β_1	β_2	β_3	β_4	β_5
U	107	109	1247	1622	919	959	443
V	80.8	132	1094	960	516	537	629

Pole positions (MeV):

	z_1	z_2
U	1428 – 35 ***i***	1384 – 62 ***i***
V	1425 – 21 ***i***	–

- The fits could be probably further improved to obtain better parameter sets, however, this was not the main aim of the work.

- More or less equal quality fits can be obtained for both potentials $\mathbf{U}$ and $\mathbf{V}$, but for very different parameter values. This means, that $\mathbf{U}$ can not be considered as an approximation to $\mathbf{V}$ $-$ they are basically different interactions.

- While the on-shell properties of the corresponding T-matrices can be made similar by suitable choice of the parameters, this is not true for their analytical behavior in energy, which governs the pole positions.

Conclusions

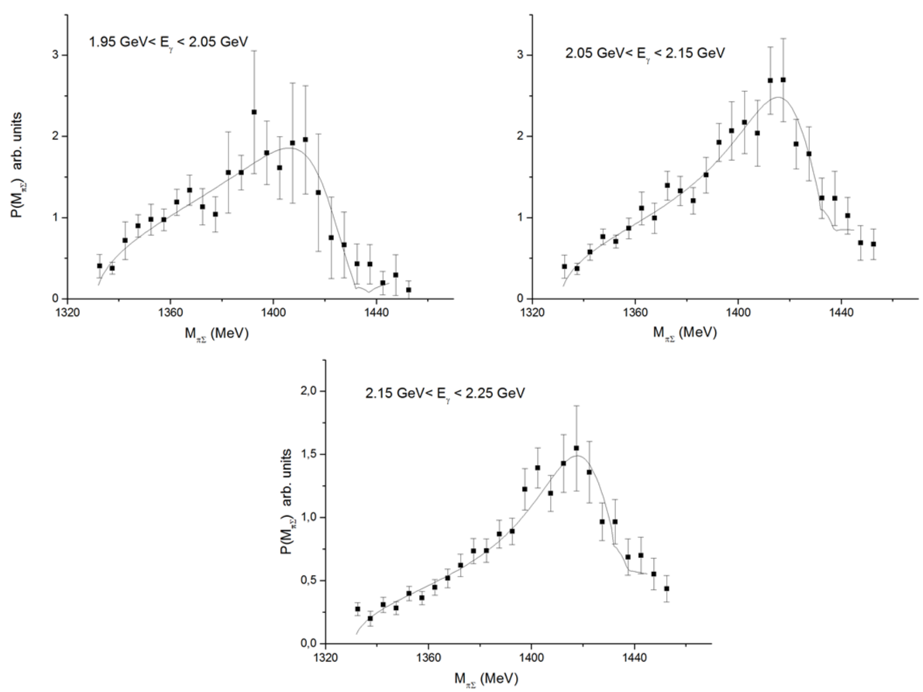
- It was shown, that the energy-dependence of the WT term of the $\overline{K}N$ interaction, derived from the chiral SU(3) Lagrangian, and responsible for the appearance of a second pole in the $\Lambda(1405)$ region, follows from the on-shell factorization approximation.

- Without this approximation a new, chiral based, energy independent $\overline{K}N$ potential was derived, which supports only one pole in the region of the $\Lambda(1405)$ resonance.

- The widely accepted "two-pole structure" of the $\Lambda(1405)$ state thus becomes questionable.

- In the calculations for $N > 2$ systems the use of the new potential avoids the difficulties arising from the energy-dependence of the $\overline{K}N$ interactions.

Most recent and accurate information on the $\Lambda(1405)$ line-shape are the CLAS photoproduction data. For their analysis at present we have the semi-phenomenological formula of Roca and Oset . It contains a few adjustable parameters and the T-matrix elements of the $\overline{K}N-\pi\Sigma$ potential. With its help we calculated the pure $I=0$ $\pi^0\Sigma^0$ missing mass spectrum for the ***unchanged*** $\mathbf{V}$ potential. The preliminary results for the lowest γ energy bins:



The complete analysis of CLAS data (including the charged states) is our next task