$a_0(980) - f_0(980)$ mixing in $\chi_{c1} \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$ and $\chi_{c1} \to \pi^0 a_0(980) \to \pi^0\pi^0\eta$

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Based On: M. Bayar, V. R. Debastiani, Phys.Lett. B775 (2017) 94-99

Outline

- Introduction
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- Results $\chi_{c1} \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$
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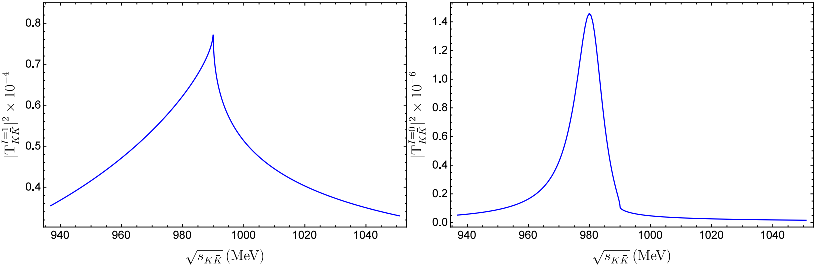
Introduction

- Both $a_0(980)$ and $f_0(980)$ have a mass around the $K\bar{K}$ threshold, and couple to $K\bar{K}$
- Their nature is still discussed, either as $q\bar{q}$, hybrids, tetraquarks and meson-meson molecules.
- Possible mixing between $a_0(980)$ (Isospin 1) and $f_0(980)$ (Isospin 0) could help to constrain models and parameters.
- Reaction $J/\psi \to \phi a_0(980)$ suggested in:
  J. J. Wu, Q. Zhao and B. S. Zou, "Possibility of measuring $a_0(980) - f_0(980)$ mixing from $J/\psi \to \phi a_0(980)$," Phys. Rev. D **75**, 114012 (2007)
  C. Hanhart, B. Kubis and J. R. Pelaez, "Investigation of $a_0 - f_0$ mixing," Phys. Rev. D **76**, 074028 (2007)

- Reaction $\chi_{c1} \to \pi^0 f_0(980)$ suggested in:
  J. J. Wu and B. S. Zou, "Study $a0(0)(980) - f(0)(980)$ mixing from $a0(0)(980) \to f(0)(980)$ transition," Phys. Rev. D **78**, 074017 (2008)
- Experimental measurement of both reactions by BESIII Collaboration:
  M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. D **83**, 032003 (2011)
- $J/\Psi \to \phi\pi^0\eta$ studied in:
  L. Roca, "Isospin violation in $J/\Psi \to \phi\pi^0\eta$ decay and the $f_0 - a_0$ mixing," Phys. Rev. D **88**, 014045 (2013)



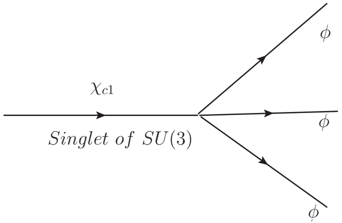
(a) $K\bar{K}$ amplitude in $I = 1$; couples to $a_0(980)$, (b) $K\bar{K}$ amplitude in $I = 0$; couples to $f_0(980)$

Figure: Comparison between $K\bar{K} \to K\bar{K}$ amplitude squared in isospin 1 and 0 (S-wave).

- $a_0(980)$ couples to $K\bar{K}$ in $I = 1$, and also to $\pi\eta$.
- $f_0(980)$ couples to $K\bar{K}$ in $I = 0$, and also to $\pi\pi$.

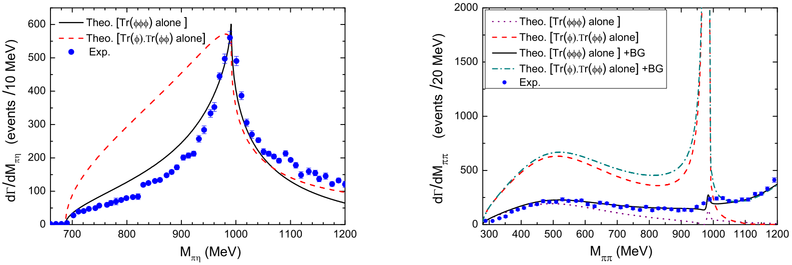
Formalism

- V. R. Debastiani, W. H. Liang, J. J. Xie and E. Oset "Predictions for $\eta_c \to \eta\pi^+\pi^-$ producing $f_0(500)$, $f_0(980)$ and $a_0(980)$," Phys. Lett. B **766**, 59 (2017).
- W. H. Liang, J. J. Xie and E. Oset, "$f_0(500)$, $f_0(980)$, and $a_0(980)$ production in the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction," Eur. Phys. J. C **76**, no. 12, 700 (2016)



- There are three independent SU(3) scalars from $\phi\phi\phi$: $Trace(\phi\phi\phi)$, $Trace(\phi)Trace(\phi\phi)$, $[Trace(\phi)]^3$

Formalism



Results for the $\pi\eta$ (right) and the $\pi^+\pi^-$ (left) mass distribution in the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction. Data from Ref. M. Ablikim, et al., BESIII Collaboration, Phys.Rev. D95 (2017) no.3, 032002.

V. R. Debastiani, W. H. Liang, J. J. Xie and E. Oset “Predictions for $\eta_c \to \eta\pi^+\pi^-$ producing $f_0(500)$, $f_0(980)$ and $a_0(980)$,” Phys. Lett. B **766**, 59 (2017).

Therefore we select:

$$\mathrm{Trace}(\phi\phi\phi) = \sqrt{3}\,\pi^0\pi^0\eta + \frac{\pi^0}{\sqrt{2}}(3\,K^+K^- - 3\,K^0\bar{K}^0), \qquad (1)$$

(we have neglected the η' components $\Longrightarrow$ large mass and small couplings to these scalar mesons)

with

$$\phi \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}. \qquad (2)$$

The next step $\Rightarrow$ let these mesons interact in coupled channels

- Isospin-allowed: $\chi_{c1} \to \pi^0 a_0(980) \to \pi^0\pi^0\eta$
- Isospin-forbidden: $\chi_{c1} \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$
- The quantum numbers of the $\chi_{c1} \Rightarrow I^G(J^{PC}) = 0^+(1^{++})$
  $a_0(980) \Rightarrow 1^-(0^{++})$
  $f_0(980) \Rightarrow 0^+(0^{++})$
- If the $\pi^0\eta$ and $\pi^+\pi^-$ are in S-wave to create the $a_0(980)$ and $f_0(980)$, the remaining π^0 must be in P-wave to conserve angular momentum and parity.

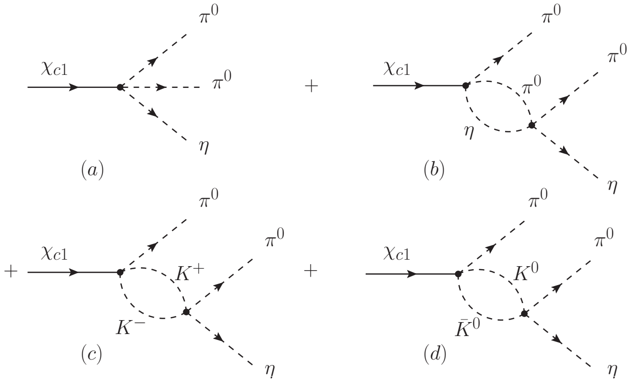


Figure: Diagrams involved in the $a_0(980)$ production in the $\chi_{c1} \to \pi^0 a_0(980) \to \pi^0\pi^0\eta$ reaction: (a) tree-level; and rescattering of (b) $\pi^0\eta$, (c) K^+K^-, (d) $K^0\bar{K}^0$.

- For $a_0(980)$ $(\pi^0\eta)$ production we have a structure at tree-level like

$$t = V_p \, \vec{\epsilon}_{\chi_{c1}} \cdot \vec{p}_{\pi^0}$$

- the full amplitude for the isospin-allowed $a_0(980)$ production (with final state $\pi^0\eta$)

$$t = \vec{\epsilon}_{\chi_{c1}} \cdot \vec{p}_{\pi^0} \, \tilde{t}_{\pi^0\eta}$$

with

$$\begin{aligned} \tilde{t}_{\pi^0\eta} &= V_p \, (h_{\pi^0\eta} + h_{\pi^0\eta} \, G_{\pi^0\eta} \, t_{\pi^0\eta\to\pi^0\eta} \\ &+ h_{K^+K^-} \, G_{K^+K^-} \, t_{K^+K^-\to\pi^0\eta} \\ &+ h_{K^0\bar{K}^0} \, G_{K^0\bar{K}^0} \, t_{K^0\bar{K}^0\to\pi^0\eta}) \, , \end{aligned} \qquad (3)$$

V_p $\Rightarrow$ a constant coefficient related to the basic dynamics of $\chi_{c1} \to$ three mesons.

The weights h_i: $h_{\pi^0\eta} = 2\sqrt{3}$, $h_{K^+K^-} = 3/\sqrt{2}$, $h_{K^0\bar{K}^0} = -3/\sqrt{2}$.

G is the loop-function

$$G_l = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2-m_1^2+i\epsilon}\frac{1}{(P-q)^2-m_2^2+i\epsilon}$$

The loop functions are regularized with a cutoff $q_{max} \sim 600$ MeV. After the integration in q^0 and $\cos\theta$:

$$G = \int_0^{q_{max}} \frac{\mathbf{q}^2 dq}{(2\pi)^2}\frac{\omega_1+\omega_2}{\omega_1\omega_2[(P^0)^2-(\omega_1+\omega_2)+i\epsilon]},$$

$$\omega_i = \sqrt{\mathbf{q}^2+m_i^2}, \quad (P^0)^2 = s.$$

Finally, for the case of $a_0(980)$ production we can write the invariant mass distribution as

$$\frac{d\Gamma}{dM_{\rm inv}(\pi^0\eta)} = \frac{1}{(2\pi)^3}\frac{1}{4M_{\chi_{c1}}^2}\frac{1}{3}p_{\pi^0}^2\; p_{\pi^0}\; \tilde{p}_\eta|\tilde{t}_{\pi^0\eta}|^2,$$

where

$$p_{\pi^0} = \frac{\lambda^{1/2}(M_{\chi_{c1}}^2, m_{\pi^0}^2, M_{\rm inv}^2(\pi^0\eta))}{2M_{\chi_{c1}}}, \quad \tilde{p}_\eta = \frac{\lambda^{1/2}(M_{\rm inv}^2(\pi^0\eta), m_{\pi^0}^2, m_\eta^2)}{2M_{\rm inv}(\pi^0\eta)}.$$

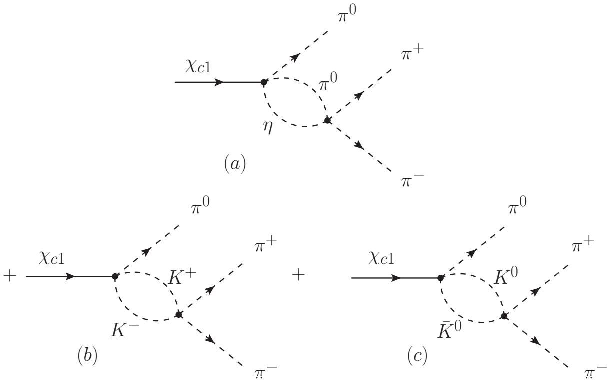


Figure: Diagrams involved in the $f_0(980)$ production in the $\chi_{c1} \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$ reaction: rescattering of (a) $\pi^0\eta$, (b) K^+K^-, (c) $K^0\bar{K}^0$.

We have no tree-level, and we consider only the rescattering diagrams

$$t = \vec{\epsilon}_{\chi_{c1}} \cdot \vec{p}_{\pi^0} \; \tilde{t}_{\pi^+\pi^-} , \tag{4}$$

$$\begin{aligned} \tilde{t}_{\pi^+\pi^-} &= V_p \, (h_{\pi^0\eta} \; G_{\pi^0\eta} \; t_{\pi^0\eta \to \pi^+\pi^-} \\ &+ h_{K^+K^-} \; G_{K^+K^-} \; t_{K^+K^- \to \pi^+\pi^-} \\ &+ h_{K^0\bar{K}^0} \; G_{K^0\bar{K}^0} \; t_{K^0\bar{K}^0 \to \pi^+\pi^-}) \, . \end{aligned}$$

For the case of $f_0(980)$ production, the invariant mass distribution:

$$\frac{d\Gamma}{dM_{\rm inv}(\pi^+\pi^-)} = \frac{1}{(2\pi)^3} \frac{1}{4M^2_{\chi_{c1}}} \frac{1}{3} p^2_{\pi^0} \; p_{\pi^0} \; \tilde{p}_{\pi^+} |\tilde{t}_{\pi^+\pi^-}|^2, \tag{5}$$

with

$$p_{\pi^0} = \frac{\lambda^{1/2}(M^2_{\chi_{c1}}, m^2_{\pi^0}, M^2_{\rm inv}(\pi^+\pi^-))}{2M_{\chi_{c1}}}, \;\; \tilde{p}_{\pi^+} = \frac{\lambda^{1/2}(M^2_{\rm inv}(\pi^+\pi^-), m^2_{\pi^+}, m^2_{\pi^-})}{2M_{\rm inv}(\pi^+\pi^-)} . \tag{6}$$

We introduce isospin violation from two sources:

- From $G_{K^+K^-}$ and $G_{K^0\bar{K}^0}$
  first loops of rescattering of K^+K^- and $K^0\bar{K}^0$ pairs
  $\Rightarrow (K^+K^- \to \pi^+\pi^-)$- $(K^0\bar{K}^0 \to \pi^+\pi^-) \neq 0$
- From the T matrix, $T = (1 - VG)^{-1}V$
  to obtain the scattering and transition amplitudes $t_{i\to\pi^0\eta}$ and $t_{i\to\pi^+\pi^-}$
  $\Rightarrow \pi^0\eta \to \pi^+\pi^- \neq 0$

We use average pion masses

the effect of using different pion masses $\Longrightarrow$ inside the T matrix and/or $G_{\pi^0\eta}$ $\Longrightarrow$ negligible!!!

Results
$\chi_{c1} \to \pi^0 a_0(980) \to \pi^0\pi^0\eta$

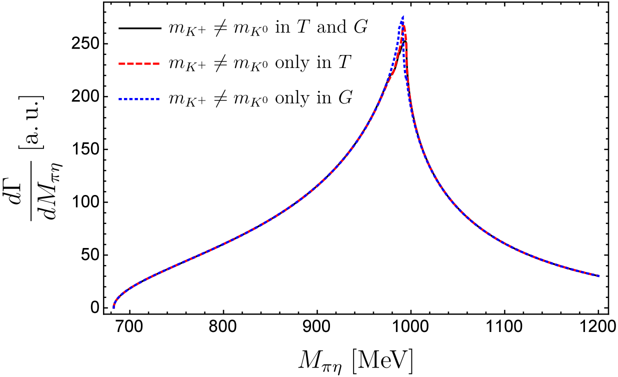


Figure: Invariant mass distribution of $\pi^0\eta$ in the $\chi_{c1} \to \pi^0 a_0(980) \to \pi^0\pi^0\eta$ reaction.

- The shape of the $a_0(980)$ is clear and the effect of isospin violation is small (this is isospin-allowed!).

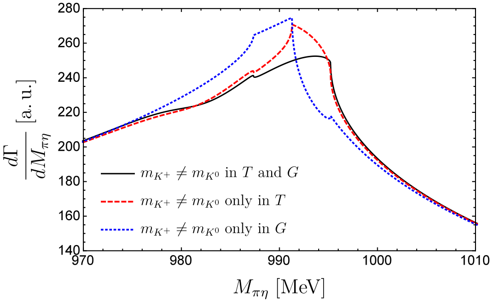


Figure: Zoom around the $a_0(980)$ peak in the invariant mass distribution of $\pi^0\eta$ in the $\chi_{c1} \to \pi^0 a_0(980) \to \pi^0\pi^0\eta$ reaction.

- In the three curves there is a small cusp effect in $M_{\pi\eta}$ at $2\,m_{K^+}$ and $2\,m_{K^0}$
- In the dashed and dotted line, where the isospin-average kaon mass $\langle m_K \rangle$ is also used, the $a_0(980)$ peak appears at $2\,\langle m_K \rangle$

Results
$\chi_{c1} \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$

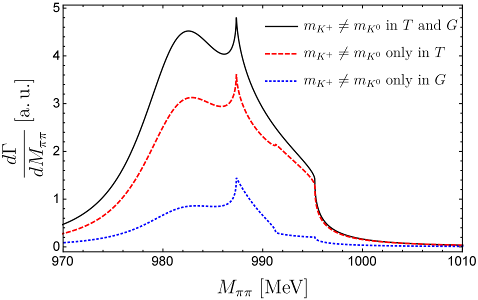


Figure: Invariant mass distribution of $\pi^+\pi^-$ in the $\chi_{c1} \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$ reaction.

- The $f_0(980)$ production is concentrated around $2\,m_{K^+}$ and $2\,m_{K^0}$
- In the dashed and dotted line we also see a cusp effect at $M_{\pi\pi} = 2\,\langle m_K \rangle$
- The bump around 980 MeV $\Rightarrow$ "good" $f_0(980)$

$\chi_{c1} \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$

- However, we have found that the isospin breaking inside the T matrix is more important than in $G_{K^+K^-}$ and $G_{K^0\bar{K}^0}$. Why? We made the following comparison:
- Contribution of $\pi^0\eta \to \pi^+\pi^-$

$$\sqrt{3}\,\pi^0\pi^0\eta \;\Rightarrow\; h_{\pi^0\eta}\; G_{\pi^0\eta}\; t_{\pi^0\eta\to\pi^+\pi^-} = 2\sqrt{3}\; G_{\pi^0\eta}\; t_{\pi^0\eta\to\pi^+\pi^-}\,,$$

- Contribution of $K\bar{K} \to \pi^+\pi^-$

$$3\,\pi^0\frac{(K^+K^- - K^0\bar{K}^0)}{\sqrt{2}} \;\Rightarrow h_{K^+K^-}\; G_{K^+K^-}\; t_{K^+K^-\to\pi^+\pi^-} + h_{K^0\bar{K}^0}\; G_{K^0\bar{K}^0}\; t_{K^0\bar{K}^0\to\pi^+\pi^-} = \frac{3}{\sqrt{2}} G_{K^+K^-}\; t_{K^+K^-\to\pi^+\pi^-} - \frac{3}{\sqrt{2}} G_{K^0\bar{K}^0}\; t_{K^0\bar{K}^0\to\pi^+\pi^-}\,.$$

- Then we compare the amplitude square of these two terms.

$\chi_{c1} \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$

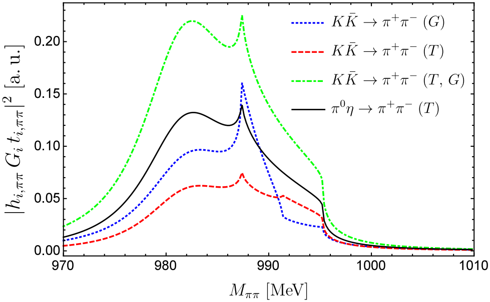


Figure: Comparison between isolated contribution of $\pi^0\eta \to \pi^+\pi^-$ and $K\bar{K} \to \pi^+\pi^-$.

- The main contribution of $K\bar{K} \to \pi^+\pi^-$ comes indeed from $G_{K^+K^-}$ and $G_{K^0\bar{K}^0}$
- But isospin breaking inside the T matrix $\Rightarrow \pi^0\eta \to \pi^+\pi^- \neq 0$

  $\Rightarrow$ ($\pi^0\eta \to \pi^+\pi^- + K\bar{K} \to \pi^+\pi^-$ only in T) $>$ ($K\bar{K} \to \pi^+\pi^-$ only in G)
- This is usually neglected by the assuming the same couplings to K^+K^- and $K^0\bar{K}^0$

Finally we calculate the $a_0(980) - f_0(980)$ mixing as the ratio of the integrated mass distributions of both reactions.

Table: Comparison between experiment and theoretical results for the $a_0(980) - f_0(980)$ mixing in the $\chi_{c1} \to \pi^0\pi^+\pi^-$ and $\chi_{c1} \to \pi^0\pi^0\eta$ reactions.

	$\Gamma(\chi_{c1} \to \pi^0\pi^+\pi^-) \, / \, \Gamma(\chi_{c1} \to \pi^0\pi^0\eta)$
BESIII [1]	$(0.31 \pm 0.16(\mathrm{stat}) \pm 0.14(\mathrm{sys}) \pm 0.03(\mathrm{para}))\%$
$m_{K^+} \neq m_{K^0}$	$M_{\pi\eta} \in [885, 1085]$ MeV
in T and G	0.26 %
only in T	0.19 %
only in G	0.05 %
$m_{K^+} \neq m_{K^0}$	$M_{\pi\eta} \in [m_\pi + m_\eta, 1200 \text{ MeV}]$
in T and G	0.17 %
only in T	0.12 %
only in G	0.03 %

[1] M. Ablikim et al. [BESIII Collaboration], "Study of $a_0(980) - f_0(980)$ mixing," Phys. Rev. D 83, 032003 (2011).

Conclusion

- it is possible to use the same theoretical model (the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction) to study the isospin breaking in the decays $\chi_{c1} \to \pi^0\pi^+\pi^-$ and $\chi_{c1} \to \pi^0\pi^0\eta$ and its relation to the $a_0(980) - f_0(980)$ mixing
- The isospin violation $\Longrightarrow$ different masses for the charged and neutral kaons: in the first propagators $G_{K^+K^-}$ and $G_{K^0\bar{K}^0}$ and inside the T matrix.
- Isospin breaking inside the T matrix: $\pi^0\eta \to \pi^+\pi^- \neq 0$
- Good agreement with the experimental data.

We would like to thank E. Oset for suggesting the topic and for the fruitful discussions.

THANK YOU FOR YOUR ATTENTION